

# A/A\* - Probability WITHOUT replacement



[www.bit.do/AgradeProbability](http://www.bit.do/AgradeProbability)

Question	Maximum Mark	Mark Awarded
1	5	
2	5	
3	3	
4	3	
5	4	
Total Mark		



1.

A box contains 2 mango yogurts, 4 vanilla yogurts and 6 cherry yogurts.  
Three yogurts are selected at random from the box without replacement.

(a) Calculate the probability that all three selected yogurts are vanilla. [2]

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(b) Calculate the probability that at least one of the selected yogurts is a cherry yogurt. [3]

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2.

Each of the numbers 1, 3, 5, 5, 5, 6, 7, 8 is written on a card.



Two of the eight cards are selected at random, without being replaced.

Find the probability that

- (a) the product of the numbers on the two cards selected is 25, [2]

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- (b) the sum of the numbers on the two cards selected is less than 15. [3]

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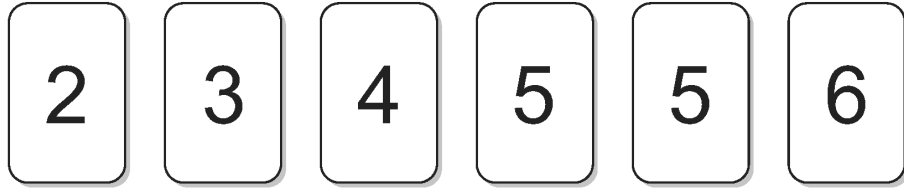
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3.

Each of the numbers 2, 3, 4, 5, 5 and 6 is written on a card.



Two of the six cards are selected at random, **without** being replaced.

Find the probability that the **sum** of the numbers on the two cards is less than 11. [3]

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4.

A bag contains four red counters and four yellow counters.  
Three counters are picked from the bag at random, **without** being replaced.

Find the probability that the three counters picked are of the same colour. [3]

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5.

A box contains 20 marbles, of which 2 are red, 3 are yellow and 15 are black.  
Two marbles are selected at random, without replacement, from the box.  
What is the probability that exactly one of the marbles is black?

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[4]

# Marking Scheme

1.

Methods in Mathematics June 2015 Unit 1 Higher Tier	Mark	Comment
18.(a) $4/12 \times 3/11 \times 2/10$ $= 24/1320 (= 1/55)$	M1 A1	Or equivalent. Ignore incorrect cancelling Or equivalent complete strategy Seen alone not part of further probabilities. May be seen as part of $1 - P(\text{no cherry})$ OR full alternative with correct values. CAO. Ignore incorrect cancelling in final answer
(b) $1 - P(\text{no cherry})$ $P(\text{no cherry}) = 6/12 \times 5/11 \times 4/10 (= 120/1320 = 1/11)$	B1 M1	
$1200/1320 (= 120/132 = 10/11)$	A1 5	

2.

15. (a) $3/8 \times 2/7$ $6/56 (= 3/28)$	M1 A1	Do not ignore incorrect cancelling  <i>OR two-way table drawn, with <math>8 \times 8 - 8</math> spaces</i> M1 $6/56 (= 3/28)$ A1  If M0A0, award SC1 for method 'with replacement' $(3/8 \times 3/8 =) 9/64$
(b) $1 - P(7 \text{ and } 8)$ $1/8 \times 1/7 + 1/8 \times 1/7$ OR $2 \times 1/8 \times 1/7$ $(1 - 2/56 =) 54/56 (= 27/28)$	S1 M1 A1	Award even if 7,8 and 8,7 are not both considered Must have considered both 7,8 and 8,7 Do not ignore incorrect cancelling  <i>OR two-way table drawn, with <math>8 \times 8 - 8</math> spaces</i> M1 $1 - P(7 \text{ and } 8)$ S1 $(1 - 2/56 =) 54/56 (= 27/28)$ A1  If M0A0, award SC1 for method 'with replacement' $(1 - 2 \times 1/8 \times 1/8 - 1/8 \times 1/8 =) 61/64$

3.

$1 - P(5 \text{ and } 6)$  $1 - [2/6 \times 1/5 + 1/6 \times 2/5]$  $26/30 (= 13/15)$ ISW	M1  M1  A1	Accept without considering different orderings.   If no other marks, award SC1 for an answer of 31/36 (from working 'with replacement') OR SC1 for sight of $[2/6 \times 1/5 + 1/6 \times 2/5]$  <i>Alternative method:</i> $P(2,3 \text{ or } 3,2 \text{ or } 2,4 \text{ or } 4,2 \text{ or } 2,5 \text{ or } 5,2 \text{ or } 2,6 \text{ or } 6,2 \text{ or } 3,4 \text{ or } 4,3 \text{ or } 3,5 \text{ or } 5,3 \text{ or } 3,6 \text{ or } 6,3 \text{ or } 4,5 \text{ or } 5,4 \text{ or } 4,6 \text{ or } 6,4 \text{ or } 5,5)$ M1 $= 2 \times 1/6 \times 1/5 + 2 \times 1/6 \times 1/5 + 2 \times 1/6 \times 2/5 + 2 \times 1/6 \times 1/5 + 2 \times 1/6 \times 1/5 + 2 \times 1/6 \times 2/5 + 2 \times 1/6 \times 1/5 + 2 \times 1/6 \times 2/5 + 2 \times 1/6 \times 1/5$ $+ 2/6 \times 1/5$ or equivalent M1 $= 26/30 (= 13/15)$ ISW A1  <i>If no marks awarded,</i> SC1 for this (alternative) method leading to 14/30 (omitting the x2 throughout) or 13/30 (not considering ordering) SC1 for this method and related answer, having omitted up to two products.  <i>OR two-way table drawn, with <math>6 \times 6 - 6</math> spaces</i> M1 $1 - P(5 \text{ and } 6)$ M1 $(1 - 4/30 =) 26/30 (= 13/15)$ A1
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4.

<p>15. <math>4/8 \times 3/7 \times 2/6</math></p> <p><math>4/8 \times 3/7 \times 2/6 \times 2</math> or <math>4/8 \times 3/7 \times 2/6 + 4/8 \times 3/7 \times 2/6</math></p> <p><math>48/336 (=1/7)</math></p>	<p>B1</p> <p>M1</p> <p>A1</p> <p>3</p>	<p>Do not ignore incorrect cancelling</p> <p>If no other marks awarded, SC1 for method 'with replacement'</p> <p><math>4/8 \times 4/8 \times 4/8 \times 2</math></p> <p>or <math>4/8 \times 4/8 \times 4/8 + 4/8 \times 4/8 \times 4/8</math> (=1/4)</p>
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5.

<p>12. Deciding on a strategy, either tree diagrams or appropriate terms</p> <p><math>P(BB') + P(B'B)</math> or equivalent</p> <p><math>\frac{15}{20} \times \frac{5}{19} + \frac{5}{20} \times \frac{15}{19}</math> must show no replacement</p> <p>150/380 or equivalent</p>	<p>S1</p> <p>M1</p> <p>m1</p> <p>A1</p>	<p>Or equivalent</p> <p>(For information: <math>P(YB)=45/380</math>, <math>P(RB)=30/380</math>)</p> <p>ISW</p>
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## Examiner's Comments

1. A few candidates seemed to only have very basic knowledge of probability.

Part (a) was often attempted with some understanding, but part (b) was beyond the knowledge of many candidates.

*This comment originally referred to question 18 on paper 4363/02 (21/05/2015)*

2. (a) This question produced answers of variable quality. Successful candidates often just wrote the relevant product of fractions and resulting answer, which was concise and perfectly adequate. Correct solutions sometimes involved the construction and use of an appropriate tree diagram, and, though not strictly necessary, this did seem to prompt a correct method. A surprising number of candidates successfully used a two-way table to illustrate the sample space, deleting the diagonal to account for the selected cards not being replaced. (Note that the two-way table would not be appropriate if selecting more than two cards.)  
  
(b) The majority of candidates realised that the most efficient method was to subtract the probability of selecting cards numbered 7 and 8 from 1. Many then failed to account for the difference between selecting 7 then 8 and selecting 8 then 7. Again, some successful candidates drew a two-way table.

*This comment originally referred to question 15 on paper 4352/02 (06/01/2015)*

3. This proved very difficult for most. Rather than the more efficient method of multiplying probabilities (possibly having constructed a tree diagram), some candidates attempted to list all the possibilities (which could produce a correct final answer).

*This comment originally referred to question 16 on paper 4352/02 (11/01/2017)*

4. Only a few fully correct solutions were seen for this question. It was common to see an attempt to calculate only one of the two required probabilities (either for all red or for all yellow counters). Few realised that the calculation of each probability would be simplified significantly by cancelling the fractions before multiplying, thus avoiding working with larger numbers.

*This comment originally referred to question 15 on paper 4352/02 (01/03/2015)*



5. Many candidates started correctly with a tree diagram, to aid the decision of which probabilities are to be considered. A common error was to include two blacks as one of the necessary possible outcomes

*This comment originally referred to question **12** on paper **4352/02 (01/04/2013)***