A/A* - Spheres, Cones, Pyramids

www.bit.do/AgradeSpheresConesPyramids

Question	Maximum Mark	Mark Awarded
1	3	
2	7	
3	6	
4	8	
Total Mark		

Education Achievement Service
for South East Wales
Gwasanaeth Cyflawni Addysg
i Dde Ddwyrain Cymru

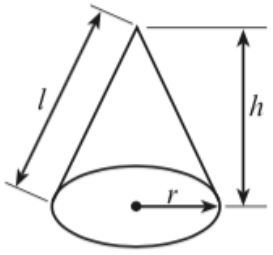
Formula booklet:

Volume of sphere $= \frac{4}{3}\pi r^3$

Surface area of sphere $= 4\pi r^2$

Volume of cone $= \frac{1}{3}\pi r^2 h$

Curved surface area of cone $= \pi r l$



1. A and B are two hollow cones.

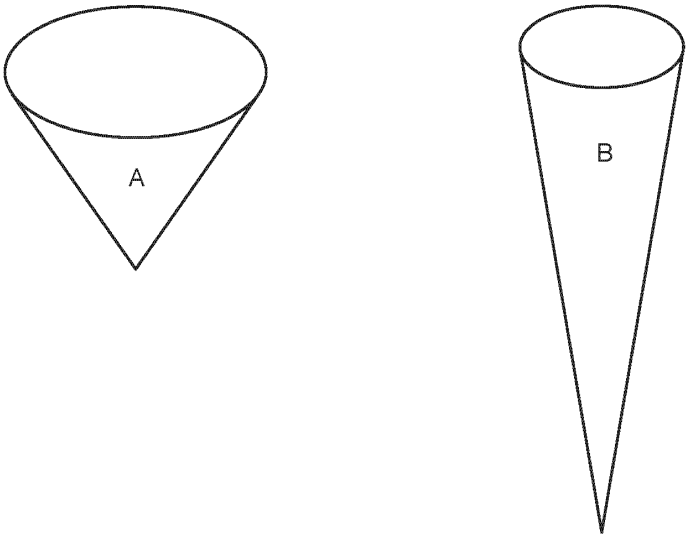


Diagram not drawn to scale

The base radius of cone B is half the base radius of cone A.
The height of cone B is twice the height of cone A.

Cone A is completely filled with water.
Is it ever possible to pour all of this water into cone B without it overflowing?
You must show working to justify your answer. [3]

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2. A hollow water container is shown below.

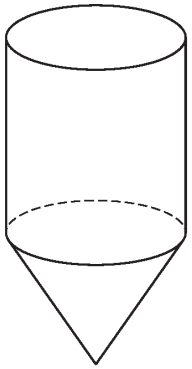

Diagram not drawn to scale

The radius of the circle formed at the join between the cone and the cylinder is 12 cm.
The height of the cylinder is five times the height of the cone.

When full, the container holds 20 litres of water.

Calculate the total height of the container.

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[7]

3. Two solid, identical spheres are attached to the ends of a solid cylinder, as shown below.

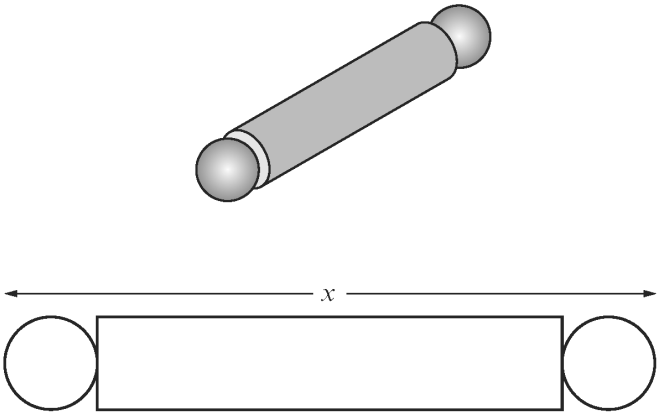

Diagram not drawn to scale

The radius, r, of each sphere is the same as the radius of the cylinder.
The length of the cylinder is $9r$.
The volume of the whole object is $3340\,\text{cm}^3$.

Calculate the total length, x, of the object. [6]

4.

Here are some facts about two solids, a square-based pyramid and a cone.

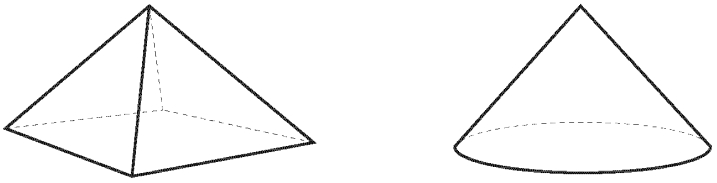

Diagram not drawn to scale

Square-based pyramid	Cone
It is a right pyramid. The total surface area of all 5 faces is $119{\cdot}8\,cm^2$. The area of **one** triangular face is $23{\cdot}6\,cm^2$. It has a volume of $76{\cdot}4\,cm^3$.	It is a right cone. It has a volume of $44{\cdot}4\,cm^3$.

The volume of each of these solids is calculated using:

$$\text{volume} = \frac{1}{3} \times \text{area of base} \times \text{perpendicular height.}$$

The square-based pyramid and the cone have equal perpendicular heights.
Calculate the radius of the cone.
Give your answer correct to an appropriate degree of accuracy. [8]

Marking Scheme

1.

			F.T. 'their consistent notation for radius and height'.
Sight of correct corresponding values for volumes. E.g. $\frac{1}{3}\pi r^2 h$ with $\frac{1}{3}\pi(r/2)^2(2h)$ or $\frac{1}{3}\pi(2r)^2 h$ with $\frac{1}{3}\pi r^2(2h)$ etc.		B2	B1 for unambiguous intent but missing brackets.
'NO' because $\frac{\pi r^2 h}{3} > \frac{\pi r^2 h}{6}$ or $\frac{4\pi r^2 h}{3} > \frac{2\pi r^2 h}{3}$		E1	Dependent on B2 AND correctly simplified in order to convincingly state 'NO'. A convincing statement required. SC2 for correctly evaluated answers using specific numerical values for radius and height of cone A and correct corresponding values for radius and height of cone B AND stating 'No'. SC1 for correct answers with no (or an incorrect) conclusion.

2.

13. (Volume of cone =) $\frac{1}{3} \times \pi \times 12^2 \times h$	✓	B1	Accept any notation or word(s) for 'height'.
(Volume of cylinder =) $\pi \times 12^2 \times 5h$	✓	B1	B1 given only when its height is noted as 5× height of cone.
20 litres = 20000 (cm^3)	✓	B1	
$20000 = \frac{1}{3} \times \pi \times 12^2 \times h + \pi \times 12^2 \times 5h$	✓	M1	F.T their '20000'. F.T. their two volumes only if of equivalent form (i.e. contains 'π' AND the height of the cylinder expressed as a multiple of the height of the cone.)
$h = \frac{3 \times 20000}{144 \times \pi \times 16}$ or equivalent	✓	A1	
$= 8{\cdot}28(93..)$	✓	A1	Accept correct to 1dp.
(Total height =) 49·7(.....) (cm)	✓	A1	F.T. 6 × 'their 8·28(93..)'.

3.

(Volume of TWO spheres =) $\frac{8\pi r^3}{3}$	✓	B1	Accept $\frac{4\pi r^3}{3} + \frac{4\pi r^3}{3}$
(Volume of cylinder =) $9\pi r^3$	✓	B1	Accept $\pi r^2 9r$.
$\frac{8\pi r^3}{3} + 9\pi r^3 = 3340$	✓	M1	F.T. 'their volumes' ONLY if dimensionally correct and 2 different shapes involved.
$r^3 = 91(\cdot 1\ldots.)$ or equivalent	✓	A1	C.A.O.
$r = 4\cdot 5$(cm)	✓	A1	F.T. from 'their r^3' if M1 awarded. Note that an unsupported answer of 4·5(cm) is zero marks.
$x = 58\cdot 5$(cm)	✓	B1	F.T. 13 × 'their stated r'

4.

17. Area of the square base = 119.8 – 4 ×23.6 = 25.4(cm^2)	M1 A1	
(Volume pyramid) 76.4 = ⅓ × 25.4 × height	m1	FT their area of square base Note √25.4 = 5.0398… × 5.0398…
height = 9.02… cm	A1*	
(Volume cone) 44.4 = ⅓ × π × r^2 × height	M1*	Depends on all previous method marks, FT their height
r^2 = 44.4/(⅓ × π × height)	M1	FT equivalent difficulty, isolating r^2
r^2 = (44.4 × ⅓ × 25.4) / (76.4 × ⅓ × π) r^2 = 4.69… to 4.701…	A1	CAO. OR r^2 = 4.7, or an appropriate unrounded r, r = 2.1681875… to 2.17
(radius) 2.17 or 2.2 (cm)	A1	CAO. Appropriate degree of accuracy required
		Alternative for A1, M1** *Equating heights,* 76.4 / (⅓ × 25.4) = 44.4 /(⅓ × π × r^2)
		For information: *Common height = 9.02(cm)*
	8	*Height of a triangular face = 9.37(cm)*

Examiner's Comments

1. To gain all three marks candidates had to show that it was never possible for the volume of cone A to be less than the volume of cone B. To do this radii and heights had to be expressed as r and h followed by r/2 and 2h or equivalent pairing.

   Nearly all candidates, however, simply took a specific set of corresponding values for the radii and heights of the cones and calculated two volumes. Some 'special case' marks were awarded for this approach if their calculations were accurate.

   *This comment originally referred to question **16** on paper **4351/02 (26/05/2016)***

2. Not well answered. Candidates are not used to having to use their own notation. Many did not have 'height' in their formula as the question had not provided them with an 'h'. Those who did introduce their own 'h' for the height of the cone then failed to realise that they should then use '5h' for the height of the cylinder. Many used 'h' again and so would not arrive at an accurate answer.

   As seen previously, candidates have difficulty in correctly manipulating formula.

   Having written $20000 = \frac{1}{3} \times \pi \times 122 \times h + \pi \times 122 \times 5h$ they were unable to arrive at

   $h = (3 \times 20000) / (144 \times \pi \times 16)$ or equivalent

   *This comment originally referred to question **13** on paper **4351/02 (01/11/2013)***

3. Not so well answered despite opportunities to gain 'follow through' marks following initial errors.

   The volume of the cylinder had to be given as $9\pi r^3$ or $\pi r^2 9r$ ($\pi r^2 h$ was not enough) before that mark was awarded.

   Also the combined volume of two spheres was required for that particular mark to be awarded.

   A follow through method mark was available for setting up an equation that was dimensionally correct.

   Once set up, however, most candidates failed to correctly manipulate their equation in order to find r^3.

   A final 'easy' independent mark was available for calculating the length of the object as 13 × 'their value for r'. Many took the length to be 11r.

   *This comment originally referred to question **15** on paper **4351/02 (26/05/2016)***

4. There are no examiner comments available for this question